Coding Theory

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Background Info

A matrix is a rectangular array of quantities or expressions in rows and columns that is treated as a single entity and manipulated according to particular rules.

$$R(90^\circ) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$W = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

The identity matrix of size n is the nxn square matrix with ones in the main diagonal and zero everywhere else.

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Coding Theory is basically sending messages from one computer to another. This means that the message can sometimes have mistakes in and hence why we have coding theory.

The codes come in binary with, for example, a 3-digit representation of a letter and a 4-digit error corrector.

Generator Matrix

The code is generally presented with a **generator matrix**, the letter representations and the message. The generator matrix is made up of the identity matrix, size nxn, and a matrix P, size nxm.

$$G = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

$$\underbrace{\qquad\qquad}_{I}\qquad\underbrace{\qquad\qquad\qquad}_{P}$$

Codewords

Codewords are important for decoding as they tell us the amount of errors that can be detected and corrected.

Codewords are found using the generator matrix. They are all the possible additional combinations of the rows. NB $\underline{0}$ is always a codeword.

Detected and Corrected Errors

The maximum number of errors detected is k-1, where k is the minimum non-zero weight of all the codewords.

The maximum number of errors corrected is the integer part of ½(k-1).

Parity Matrix

The parity matrix is a check matrix which is the P part of the generator matrix with the identity matrix written underneath it.

$$H = \left[\begin{array}{c} P \\ \hline I \end{array}\right]$$

Syndrome and Coset Leader

The syndromes are the rows of the parity matrix and also 0.

The coset leader is same length as the messages and has a 1 in the position which corresponds to the row of H. So if it’s the fourth row, it’ll have a 1 in the fourth position.

Solving Table

Received Message **w**	Syndrome **wH**	Coset Leader **h**	Corrected Message **w+h**	Result	Letter

Question Example

A group code has generator matrix

$$G=\begin{pmatrix} 1&0&0&0&1&1&1 \\ 0&1&0&1&1&0&0 \\ 0&0&1&1&0&1&0 \end{pmatrix},$$

Using the following letter to number equivalents:

M	U	D	C	R	O	W	N
000	100	010	001	110	011	101	111

Correct and read the message:
1011010, 0110010, 1110001
1100111, 1110000, 0101110
0101011, 1010111, 0100000.